[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **4801** **G**

Unique Paper Code : 42354302

Name of the Paper : Algebra

Name of the Course : **B.Sc. (Prog.)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.

Unit I

1. (a) Find the inverse of $\left\{\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right\}$ in $GL(2, Z_5)$, the group of 2 × 2 non-singular matrices over Z. Verify the answer by direct calculation. (6)

P.T.O.

(b) Describe the group of symmetries of a square and draw its Cayley's table. (6)

(c) Find the orders of each of the elements of U(14). Show that it is cyclic and find all its generators. (6)

2. (a) Let G be a group and let $a \in G$. Prove that $\langle a^{-1} \rangle = \langle a \rangle$. (6)

(b) State and prove Lagrange's Theorem. (6)

(c) If G is a group such that $x^2 = e$ for all elements x of G where e is the identity element of G then prove that G is an abelian group. (6)

3. (a) Prove that in a finite group, the order of each element of the group divides the order of the group. (6)

(b) Define an alternating group. Find all the elements of A_4. (6)

(c) Let $\sigma = (1,5,7)(2,5,3)(1,6)$. Then find σ^{17}. (6)

Unit II

4. (a) (i) Let a belong to a ring R. Let $S = \{x \in R : ax = 0\}$. Show that S is a subring of R.

(ii) Prove or disprove $3Z \cup 5Z$ is a subring of the ring Z of integers. (6.5)

(b) State the Subring Test and Show that the set

$$S=\left\{\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} : x \in R\right\}$$ is subring of the ring of all 3×3 matrices over real numbers. Also find the unity of S. (6.5)

(c) Define an ideal of a ring R. Prove that the intersection of two ideals of a ring R is an ideal of a ring R. What can you say about the union of two ideals of a ring R? Justify. (6.5)

Unit III

5. (a) Prove that a non - empty subset W of a vector space V(F) is a subspace of V if and only if $\alpha x + \beta y \in W$ for all $\alpha, \beta \in F$ and $x, y \in W$. (6.5)

P.T.O.

(b) Let {a, b, c} be a basis for the Vector Space. Prove that the set {a+b, b+c, c+a}, {a, a+b, a+b+c} are also bases of R^3. (6.5)

(c) Define Linear Transformation. Check whether the mapping T: $R^2 \rightarrow R^2$ defined by T(x, y) = (1 + x, y) is a Linear Transformation. (6.5)

6. (a) Suppose that T: $R^2 \rightarrow R^2$ is a Linear Transformation. If T (1,0) = (1,4) and T (1,1) = (2,5). Find T (2,3). Is T one-to-one? (6.5)

(b) Let T: V -> U be a Linear Transformation. Then prove that T is one-to-one if and only if the null space N(T) = {0}. (6.5)

(c) Let T: $R^2 \rightarrow R^3$ be a Linear Transformation defined by T(x,y) = (x, x+y, y). Find the Range, Rank, Kernal and Nullity of T. (6.5)